



Robustness on the *R&D* alliance network: A multi-layer approach

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Outline of talk

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- Two-layer approximation

- Multi-layer network

Introduction

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Multi-layer network

Motivation

- ▶ Understanding the **robustness** of a network against removing nodes involves **analyzing its topology** and plays a significant role in **improving its resilience**.
- ▶ Most researches focus on **robustness analysis** in **unipartite** graphs.
- ▶ Exceptions for **two-layer network*** reveal how **differentiating** nodes in different layers and **multi-layer** structure influence the robustness of the network.
 - ▶ **Inter-layer** connections.
 - ▶ **Intra-layer** connections.
- ▶ What about the **robustness** of **multi-layer** networks with **three or more layers**?

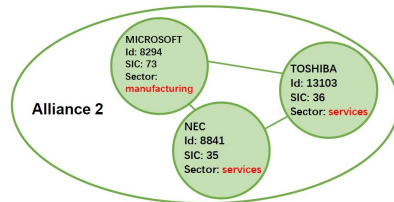
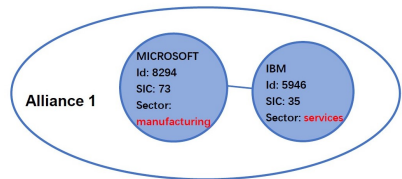
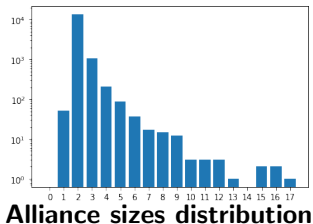
*casiraghi2019probing.

Empirical dataset

- ▶ 14535 valid **firms**, each firm belongs to a sector.
- ▶ 10 **sectors**: **services**, **manufacturing**, **public administration**...
- ▶ 14881 valid **alliances** with **participated firms** and durations.
 - ▶ Each firm can participate in **several alliances**.

	Minimum	Median	Maximum	Mean
#alliances per firm	1	1	284	2.2017

- ▶ Alliance size: **any positive number**.



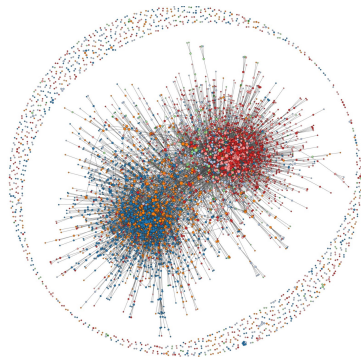
Dataset snapshot

Firm network

- ▶ **Node** \leftrightarrow **Firm**; **Layer** \leftrightarrow **Sector**.
- ▶ An **undirected** and **unweighted edge** exists between two firm nodes if they share **common alliance(s)**.

	Minimum	Median	Maximum	Mean
#node degree	0	1	421	2.9643

- ▶ **Static network**: **durations** of alliances are **ignored**.
- ▶ **Periphery**: not in the largest connected component (**LCC**) of the network.
 - ▶ **Periphery fraction**: 41.07%.
- ▶ A **similar topology** to the network[†] on the right.



[†]Giacomo2019.

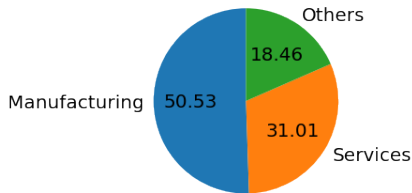
Characterising properties

- ▶ **Large scale:** 14535 nodes.
 - ▶ Inefficient to analyse some properties such as the nestedness.

- ▶ **Sparsity:** for matrix M

$$\text{Filling rate}(M) = \frac{\# \text{non-zero entries}}{\# \text{row} \times \# \text{column}} \quad (1)$$

- ▶ **filling rate** of the adjacency matrix is 0.000188.
- ▶ **Dominant layers:** **manufacturing** layer and **services** layer.
 - ▶ In the **whole network**, the **fraction of nodes** in these two layers are above 80%.
 - ▶ In the **LCC**, the **fraction of nodes** in these two layers: 84.71%, the **fraction of edges** involving nodes in these two layers: 94.15%.



Node-layer distribution

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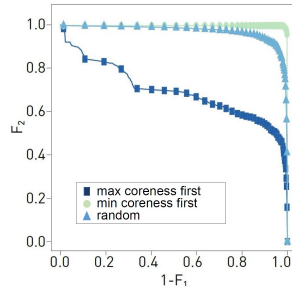
Results

Two-layer approximation

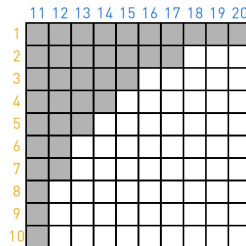
Multi-layer network

Cascading model[‡]

- ▶ **Attacked layer:** its nodes are removed with three strategies.
 - ▶ Remove nodes **randomly** (random), with **increasing coreness** (min), and with **decreasing coreness** (max).
- ▶ **Synthetic two-layer network:**
 - ▶ A node is removed when disconnected with the **attacked layer**.
 - ▶ **Coupling: inter-layer** connections.
 - ▶ **Nestedness:** “Nestedness can be described as the **tendency** for nodes to interact with subsets of the interaction partners of **better-connected nodes**”.
- ▶ **Multi-layer network**
 - ▶ A node is removed when disconnected with **all the other layers**.



Cascades results



Perfectly nested incidence matrix
“Generalists” to “generalists”

[‡]casiraghi2019probing.

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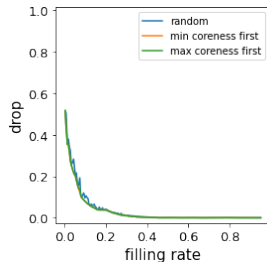
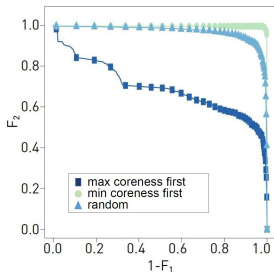
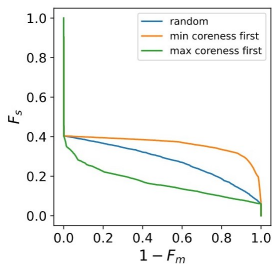
Results

Two-layer approximation

Multi-layer network

The importance of filling rate

- ▶ Big “drops” in the empirical network, not in the **baseline synthetic network**.
- ▶ Empirical network: filling rate of the **incidence matrix** $\rightarrow 0.000326$.
- ▶ Baseline synthetic network: filling rate of the **incidence matrix** $\rightarrow 0.35$.



Empirical network (LCC)

- ▶ The drops are caused by the **low filling rate** and the **assumption that nodes get removed once disconnected with the attacked layer**.
- ▶ The drop $\rightarrow 0$ when filling rate $\gtrapprox 0.35$.

Synthetic network

Filling rates and “drops”

Introduction

Method

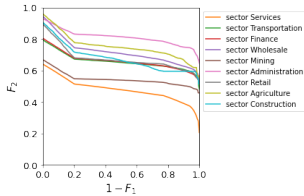
Results

Two-layer approximation

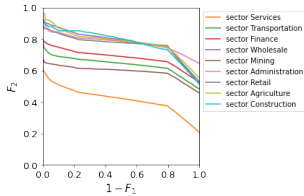
Multi-layer network

Cascades in multi-layer network

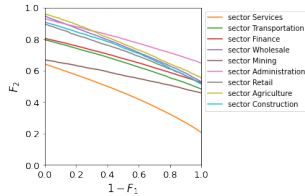
- ▶ An example of cascade results on the **whole network**.
- ▶ Attacked layer is the **manufacturing** layer (the **largest layer**, **50.53%**).
- ▶ “Drops” at the beginning \leftrightarrow Sparsity (filling rate of the adjacency matrix is **0.000188**).



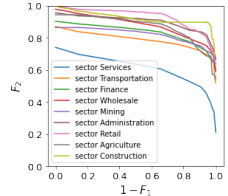
min coreness first (whole)



max coreness first (whole)



random (whole)



min coreness first (LCC)

- ▶ **Robustness**: even attacking the largest layer (**half** of the nodes) doesn't cause huge cascades.
- ▶ **Min coreness first**:
 - ▶ **Fragile periphery**: **linear faster decays** at the beginning in the whole network scenario.
 - ▶ **Resistance of LCC**: cascades **slow down** after the **turning point**.

Why is the periphery important?

► Low-coreness:

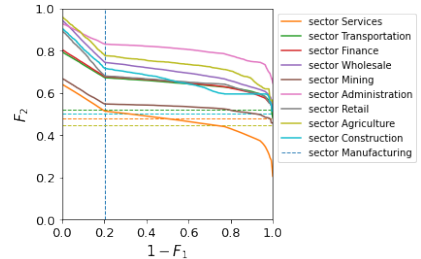
- A **considerable** part of nodes with **low coreness** in each layer are in the periphery.
 - **Coreness 0**: Manufacturing layer: **55.46%**; Services layer: **42.07%**.

► “Well-mixed” periphery:

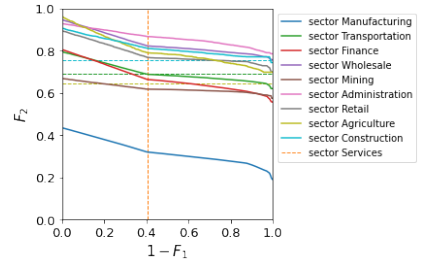
- **92.3%** of alliances in the periphery have sizes of 2.
 - **54.8%** of 2-alliances consist of two firms from the **same** sector.
 - **45.2%** of 2-alliances consist of two firms from **different** sectors.
- **87.62%** of nodes in the periphery have **coreness 1**.

► Results under “min coreness first” attacks:

- **Vertical lines**: fractions of nodes with **coreness 0**; **Horizontal lines**: fractions of nodes **connected with** the attacked layer.
- **Robustness**: most of curves are **close to** their corresponding horizontal lines.



Manufacturing layer

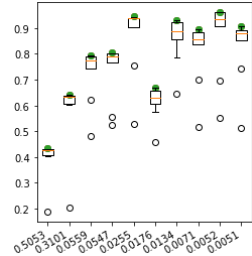


Services layer

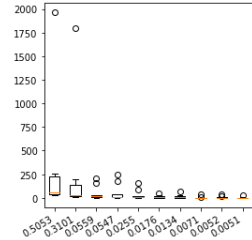
Robustness analysis

Cascade results with different attacked layers:

- ▶ **Final fractions of remaining nodes:**
 - ▶ **x-axis:** layer **proportion**, e.g. 0.5053-manufacturing layer; **y-axis:** **remaining fractions of x**, the attacked layer \in the other 9 layers; **Green scatters:** **real initial fractions** excluding “drops”.
 - ▶ **Robustness:** removing any layer **won't damage** the **majority** of the **remaining parts** of other layers.
 - ▶ **Outliers:** manufacturing layer; services layer.
- ▶ **Damages caused by the attacked layer:**
 - ▶ **x-axis:** layer **proportion** of the **attacked layer**; **y-axis:** **#removed nodes in other 9 layers (drops excluded)**.
 - ▶ **More outliers** and **decreasing interquartile ranges:** **small layers** are more **unevenly** connected with other layers.



Final fractions



Damages

Summary

► Insights into the empirical dataset

- **Multi-layer structure** **adds** to the **robustness**.
 - Protect nodes against attacks.
 - Constrain the propagation of cascades.
- **“Well-mixed” struture** of the periphery → **more fragile** under the “min coreness first” attacks.
 - **Instant huge losses** can be **avoided** if the periphery is “protected”.

► Methodology

- In **cascade simulations**, **“drops”** (cascades) can also be caused by the **sparsity** of the network.
 - These **“ drops”** should be **differentiated** from standard cascading processes.
- Focusing on the LCC is not enough: **the periphery can expose unexpected fragilities and weaknesses**.
 - Important information is **maintained** → **LCC approximation**.

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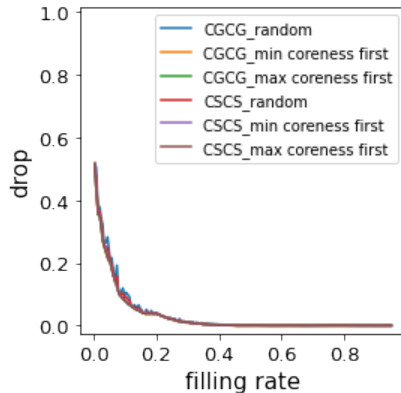
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Thanks for your time!

Q&A

Slides for possible questions

1. Alliances of size 1 are formed by several subsidiaries of one firm.
2. Filling rates-“drops” relationships under different coupling strategies.
3. “Drops” distribution:
 - ▶ 6233 (42.88%) nodes in total.
 - ▶ 48.36% of “drops” are in the periphery.
 - ▶ 51.64% of “drops” are in the LCC.



4. Nestedness analysis for large-scale and sparse network

- ▶ **Two-layer approximation + bipartite projection.**
 - ▶ Connectance of the bigraph: **0.00015**.
- ▶ **Large-scale** empirical dataset: **computational efficiency**. Popular algorithms such as NODF, MT, and BR → **computationally inefficient**.
- ▶ **Correlation** with other properties:
 - ▶ E.g. dissortativity, core-periphery metric ξ , and modularity.
 - ▶ **Modularity**: **low-density** networks exhibit a **positive correlation** between nestedness and modularity, while **high-density** networks exhibit a **negative correlation** between them.
 - ▶ The **z-score** computed with 50 null models generated by Bascompte's algorithm is **-16.6718**. The probability to reject the hypothesis that our empirical bigraph consists of "Manufacturing" layer and "Services" layer has modularity is higher than **0.999999**.
 - ▶ **No modularity w.h.p** $\xrightarrow{w.h.p}$ **no nestedness w.h.p.**